# **Robot Trajectory Planning of Space Curve**

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Abstract: Aiming at solving the difficulty problem of the trajectory planning of the cylindrical space curve, a method is proposed for reconstructing the interpolation space curve of three-dimensional coordinates of accurate target points of cylindrical surface, and the discrete continuous space curve is obtained. In order to obtain the pose change matrix of discrete curves, the forward tangential method is used to calculate the curve tangent vector. Then, the curve normal vector is calculated by using the center coordinate of the bottom surface of the cylinder and the precise target point projection of the curved surface, the curvature vector is obtained by the cross-multiplication of the right-hand rule, thereby forming a cosine matrix whose cylindrical curved surface changes direction. The three-dimensional coordinates of the center of the cylinder and the precise target point of the cylindrical surface are used as the position change vector of the measurement curve, thus constructing the homogeneous matrix of the variation curve. The MATLAB simulation shows that the pose homogeneous matrix constructed by this method has good trajectory follow-ability and attitude change continuity, which provides technical support for the next step of planning robot machining space curve.

#### 1. Introduction

As the rigidity and precision of robots increase, robots are increasingly used in the field of machining. However, the key part that the research results of the surface trajectory attitude planning problem are few, and all the methods have certain defects. Wang H F et al [1] proposed an optimization analysis of the running time and acceleration of the robot trajectory based on the NGGA-2 algorithm. Zhu S Q et al [2] used 7 B-spline curves to describe the trajectory of the robot, which satisfies the requirements of time optimum and continuous acceleration, but only meets the physical quantity requirements of the robot movement, and lacks strict calculation and analysis of the key hurdles to its trajectory posture. Song et al [3] improved the smoothness of the grinding trajectory by using the cooperative particle swarm algorithm to control the parameters of the robot. Based on the classical section method, Wang et al [4] proposed an arc length optimization algorithm for cutting trajectory planning, which further adapted the curve to meet the smoothing requirements of robot motion. In recent years, the algorithm of point cloud data reconstruction path planning path is proposed. Wang X L et al [5] used the 3D scanner to obtain the point cloud data for surface reconstruction. After obtaining the 3D model, a path generation method based on tangent plane is proposed. Masood et al [6] used the point cloud data to generate the NC machining path algorithm, using the section normal on the tool contact path and generating the machining path by tool offset. However, the trajectory accuracy of such point cloud reconstruction is insufficient, the surface fitting algorithm is complex, and the processing path continuity is poor.

In order to improve the above defects, a binocular camera tracking target is proposed, and the rotating world coordinate system is automatically calculated to complete the 360-degree panoramic trajectory reconstruction of the cylindrical object curved target. After the high-order spline fitting with precise trajectory points, the discrete pose algorithm is used to quickly generate the pose trajectory of the robot, which is verified by simulation experiments and is expected to be applied in the field of robot cutting engineering.

# 2. 3D reconstruction of target points on cylindrical surfaces

By setting up a binocular camera experimental environment, camera calibration is implemented for the platform by Halcon software. Through completing 3D reconstruction and multiple field splicing work, the trajectory extraction of target points on the surface can be completed, and the 3D coordinates and visualization model can be obtained as shown in Figure 1 and Table 1:





a Target display on a cylindrical object surface b trajectory after 3D reconstruction of target

Fig.1 Comparison of target trajectory reconstruction

Table 1 Trajectory world coordinates under six poses

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X	у	Z	X	у	Z
-4.47077	45.3343	-65.3792	17.2486	-42.1281	-62.8574
-10.7239	44.0184	-60.0996	11.1596	-44.0246	-57.4907
-15.9431	42.2474	-53.7082	23.7587	-39.0251	-66.7299
-20.2831	40.0426	-47.0697	5.67213	-44.8949	-51.3188
-24.0247	37.6547	-40.1974	30.0616	-34.1863	-69.5896
-27.3184	35.1549	-32.9508	0.782435	-45.0217	-44.5103
-30.3526	32.3436	-25.5848	-3.56485	-44.6262	-37.3661
-33.3941	28.9972	-18.3064	-7.71277	-43.7856	-30.2402
-44.3854	-0.916935	-56.6251	-12.0504	-42.4766	-23.1821
-43.9312	3.24516	-49.8801	35.9325	-28.4488	-69.5238
-43.3691	7.08254	-42.5207	30.2013	-34.3824	-69.2808
-42.6878	10.6839	-34.7013	40.2012	-21.4707	-67.972
-41.3777	14.1791	-27.5325	43.1472	-14.2493	-65.0595
-30.224	32.5304	-25.5634	23.6056	-39.0745	-67.0302
-39.8984	18.0357	-20.1913	44.8384	-7.0544	-61.4039
-33.2697	28.9925	-18.5702	45.5091	-25.1415	-56.5349
-37.1963	23.339	-14.5069	45.5765	5.39169	-50.6221
-38.7525	-21.4212	-51.5188	41.6464	18.5807	-66.2476
-36.8922	-24.092	-44.1134	43.7152	13.5538	-60.3271
-34.7071	-26.7976	-36.4278	39.1356	23.5746	-60.542
-32.058	-29.6642	-29.0198	37.4744	26.4241	-52.8585
-28.7311	-32.7802	-22.084	44.7438	10.0653	-52.8967
-23.8762	-36.392	-16.4489	35.9189	28.7844	-45.0712
0.738498	-44.8554	-44.4929	34.3538	30.9418	-36.9648
-3.58936	-44.4082	-37.4481	8.57281	44.4551	-60.5686
-7.74196	-43.6564	-30.2598	32.5519	33.0054	-29.0969
5.51852	-44.6758	-51.4987	11.3752	43.8609	-52.9607
-12.0735	-42.3674	-23.2879	13.6961	43.3275	-45.0121
-17.2287	-40.2845	-17.0578	30.2913	35.1953	-21.3872
21.3646	40.8145	-21.3756	15.9891	42.7043	-37.0399
26.0334	38.3677	-15.366	18.4254	41.8852	-29.2453

### 3. 3D target point trajectory fitting and attitude analysis

# 3.1 Spline fitting of 3D trajectory points

The target point is the key information point that the cutting path must pass. The cubic spline curve fitting can make the machining path enough continuous and smooth, and it will not cause bad behavior such as robot stagnation and self-locking. The path of the trajectory formed after fitting the target point by MATLAB software is shown in Figure 2:

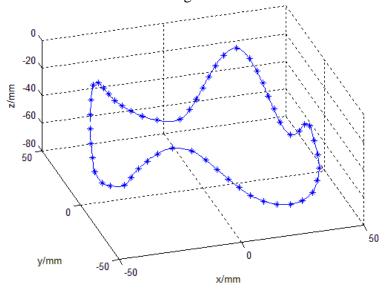


Fig.2 Track after target fitting

## 3.2 Solving the pose of the 3D trajectory

The spatial curve formed by the surface of the cylinder changes its attitude as the point coordinates change. In order to obtain the changed attitude matrix and to make sure that the subsequent robot end can effectively track the change attitude, according to the special properties of the cylindrical surface and the smooth condition of the spline curve position, a fast discrete difference algorithm is proposed to calculate the curve tangent vector. The curve normal vector is calculated by using the center coordinates of the bottom surface of the cylinder and the precise target point projection on the curved surface, and the curvature vector is obtained by cross-multiplying according to the right-hand rule. Thereby, a cosine array of directions in which the direction of the curve changes on the cylindrical surface is formed.

Among them, the solution of the tangent vector  $\mathbf{k}$  is the most critical, and the principle is shown in Figure 3.Avoiding the difficulty of solving the derivative of the fitted curve, and discretizing the curve by the distance  $\Delta t$ , and apply the forward first-order difference approximation of the discrete points to cut the tangent vector:

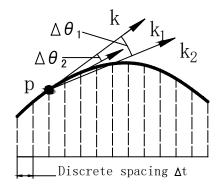


Fig.3 Discrete Difference Principle

$$\begin{cases}
K(t)_{X} = X(t+1) - X(t) \\
K(t)_{y} = Y(t+1) - Y(t) \\
K(t)_{z} = Z(t+1) - Z(t)
\end{cases}$$
(1)

The corresponding unit tangent vector is expressed as:

$$\mathbf{k} = \frac{\begin{bmatrix} \mathbf{K}(t)_{X} & \mathbf{K}(t)_{y} & \mathbf{K}(t)_{z} \end{bmatrix}^{T}}{\begin{bmatrix} \mathbf{K}(t)_{X} & \mathbf{K}(t)_{y} & \mathbf{K}(t)_{z} \end{bmatrix}^{T}}$$
(2)

When the value of the discrete pitch  $\Delta$  decreases, the angular deviation  $\Delta\theta$  between the difference vector of the point p and the true tangent vector also becomes smaller. In order to ensure the smooth transition of the tangent and the discrete algorithm is convenient and feasible,it is necessary to implement multiple dispersion between adjacent target points, that is, to sequentially divide into 2N equal parts. Through data verification, when the discrete is 10 equal parts, the value of  $\Delta \mathbf{k}$  approaches zero, and the angle of the tangent vector changes slowly, satisfying the requirement of smooth continuity.

The normal vector b can be solved by the z-axis projection method. The principle is shown in Figure 4.By using the special properties of the cylindrical object surface, the two-dimensional discrete circular coordinates are obtained by projecting the discrete points onto the XOY plane. The straight line vector formed by the projection point and the origin of the coordinate system constitutes the plane normal  $b_1$ , which is translated to the cylindrical discrete point and then upgraded into a three-dimensional normal vector b:

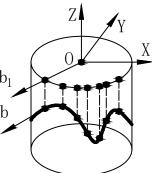


Fig.4 Principle of Z-axis projection

$$\mathbf{b} = \frac{\begin{bmatrix} \mathbf{b}_1 & \mathbf{0} \end{bmatrix}^T}{\| \begin{bmatrix} \mathbf{b}_1 & \mathbf{0} \end{bmatrix}^T \|}$$
 (3)

The curvature direction  $\mathbf{m}$  is used as secondary information and can be obtained by right-hand rule:

$$\mathbf{m} = \mathbf{b} \times \mathbf{k} \tag{4}$$

By combining the above-mentioned attitudes with the discrete points in the absolute position of the coordinate system, a homogeneous matrix of the variation curve poses can be formed:

$$T = \begin{bmatrix} \mathbf{k} & \mathbf{m} & \mathbf{b} & \mathbf{p} \end{bmatrix} \tag{5}$$

The trajectory posture of the surface curve of the cylinder can be calculated from the homogeneous matrix of (5), and simulated by MATLAB. The results are shown in Figure 5 below. Its normal vector b is perpendicular to the surface curve of the cylinder and follows the position point, which provides a basis for planning the change attitude of the end tracking curve of the robot.

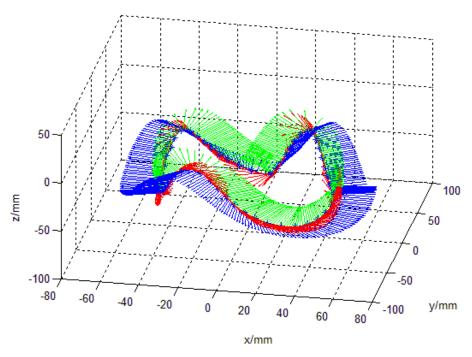


Fig.5 Discrete track pose

#### 4. Conclusion

By constructing the tracking pose square matrix target at the bottom of the cylinder, the three-dimensional coordinate reconstruction of the trajectory target point on the cylinder surface is realized, and the three-dimensional coordinates of the target point trajectory are obtained. Then it construct the tangent matrix, curvature vector, normal vector and positional change matrix of the change with the position of the curve. Through MATLAB simulation, the normal vector b is perpendicular to the curve on the surface of the cylinder and follows the change of position point, which provides a new method for planning the change attitude of the end tracking curve of the robot.

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